

Spacetime Topology from the Tomographic Histories Approach: Part II

Ioannis Raptis,^{1,2,5} Petros Wallden,³ and Romàn R. Zapatrin⁴

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As an inverse problem, we recover the topology of the effective spacetime that a system lies in, in an operational way. This means that from a series of experiments we get a set of points corresponding to events. This continues the previous work done by the authors. Here the relativistic case is considered. The existence of upper bound in the speed of transfer of matter and information induces a partial order on the set of events. While the actual partial order is not known in our operational set up, the grouping of events to (unordered) subsets corresponding to possible histories, is given. From this we recover the partial order up to certain ambiguities that are then classified. Finally two different ways to recover the topology are sketched and their interpretation is discussed.

KEY WORDS: quantum gravity; decoherent histories; spacetime topology.

1. INTRODUCTION

The work presented here, is a continuation of Raptis *et al.* (2005) in which we introduced the concepts of inverse histories, effective topology and operationalistic means of recovering the background structure of effective spacetime. In that paper we dealt with the non-relativistic case, while here we proceed to the relativistic one. The key difference is the existence of an upper bound in the transfer of matter and information. Due to this, the set of possible events (\mathcal{P}) has the extra structure of a partially ordered set (with respect to the causality relation). This leads to

¹EU Marie Curie Reintegration Postdoctoral Research Fellow, Algebra and Geometry Section, Department of Mathematics, University of Athens, Panepistimioupolis, Athens 157 84, Greece.

²Visiting Researcher, Theoretical Physics Group, Blackett Laboratory, Imperial College of Science, Technology and Medicine, Prince Consort Road, South Kensington, London SW7 2BZ, UK.

³Theoretical Physics Group, Physics Department, Imperial College; e-mail: petros.wallden@imperial.ac.uk.

⁴Department of Information Science, The State Russian Museum, Inzenernaya 4, 191186, St.Petersburg, Russia; e-mail: zapatrin@rusmuseum.ru.

⁵To whom correspondence should be addressed; e-mail: i.raptis@ic.ac.uk.

further restrictions on the set of possible histories/‘trajectories,’ that in its turn, results to some proximity relation from the bare set of possible histories.

1.1. Motivation

Here we should make a clarification about what the records are. Each of these, corresponds to a total (generalized) trajectory of the system and consists of many sub-records that correspond to each of the points/events that constitute this trajectory. Having got just a ‘frozen picture’ (at the final time) of the trajectory we are not able to say which event (sub-record) occurred before and which after if we do not presuppose anything about the dynamics and the background.

We now briefly remind few concepts from Raptis *et al.* (2005).

1.2. Histories and Inverse Histories

The *standard decoherent* histories approach to quantum mechanics deals with the kind of questions that may be asked about a closed system, without the assumption of wavefunction collapse (upon measurement). It tells us, in a non-instrumentalist way, under what conditions we may meaningfully talk about statements concerning histories of our system, by using ordinary logic. This approach was mainly developed by Gell-Mann and Hartle (1990a,b,c); Hartle (1991a, b, 1992, 1993), and it was largely inspired by the original work of Griffiths (1984) and Omnès (1988a,b,c, 1989, 1990, 1992). A more mathematically sophisticated formulation is due to Isham and collaborators in *e.g.*, Isham (1994).⁶ This formulation, consists of a space of histories \mathcal{UP} , which is the space of all possible histories of the closed system in question, and a space of decoherence functionals \mathcal{D} . Parenthetically, the space of histories is usually assumed to be a tensor product of copies of the standard Quantum Mechanics’ Hilbert space. Two histories are called disjoint, write $\alpha \perp \beta$, if the realization of the one excludes the other. Two disjoint histories can be combined to form a third one $\gamma = \alpha \vee \beta$ (for $\alpha \perp \beta$). A complete set of histories is a set $\{\alpha_i\}$ such that $\alpha_i \perp \alpha_j$ ($\forall \alpha_i, \alpha_j, i \neq j$), and $\alpha_1 \vee \alpha_2 \vee \dots \vee \alpha_i \dots = \mathbf{1}$.

A decoherence functional is a complex valued function $d : \mathcal{UP} \times \mathcal{UP} \rightarrow \mathbb{C}$ with the following properties:

- a) Hermiticity: $d(\alpha, \beta) = d^*(\beta, \alpha)$
- b) Normalization: $d(1, 1) = 1$
- c) Positivity: $d(\alpha, \alpha) \geq 0$
- d) Additivity: $d(\alpha, \beta \oplus \gamma) = d(\alpha, \beta) + d(\alpha, \gamma)$ for any $\beta \perp \gamma$

⁶H.P.O. History Projection Operator.

A complete set of histories $\{\alpha_i\}$ is said to obey the DECOHERENCE condition, *i.e.* $d(\alpha_i, \alpha_j) = \delta_{ij}p(\alpha_i)$ while $p(\alpha_i)$ is interpreted as the probability for that history to occur *within the context of this complete set*. The decoherence functional encodes the initial condition as well as the evolution of the system. Here we should also note that the topology of the space-time is presupposed when we group histories into complete sets, *i.e.* in collections of partitions of unity.

In Quantum Mechanics, histories correspond to time ordered strings of projections and to combination of these when they are disjoint. An important issue here is the relation between decoherence and records. Namely, it can be shown that if a set of histories decoheres, there exists a set of projection operators on the final time that are perfectly correlated with these histories and vice versa.⁷ These projections are called *records*. It is this concept that figures mainly in our approach (*e.g.*, see Halliwell, 1999). Note here that in the standard (as opposed to the inverse to be introduced below) formulation of the decoherent histories there exist more than one (incompatible) decoherent set of histories, in general, and for each of these sets, a set of records exists (incompatible with those corresponding to other decoherent sets). The persistence of these records in time is something not in general guaranteed.

To sum things up, in the standard histories approach

- Both the system and its environment are given. The latter is represented by prescribing initial conditions and in some cases final conditions.
- The space, in particular, its topological structure is presupposed.
- The interactions are given in terms of the decoherence functional, which encodes the dynamical information. For the complete dynamics, the full Hamiltonian must be known.

In the inverse histories approach (or else ‘tomographic’) developed in our previous paper (Raptis *et al.*, 2005) things are different. We solve the *inverse* problem. While in standard histories we are given

- the Hamiltonian
- initial conditions
- the space on which they are defined

and the aim is to predict probabilities for histories, we do the opposite thing. We have the relative frequencies corresponding to different histories (belonging to a particular decoherent set) and we consider the records in the ‘final’ time that are related to this histories.⁸ The decoherent set the histories that we measure,

⁷This is the case for a *pure* initial state, and we restrict ourselves to it.

⁸The existence of these records is guaranteed by the relation of decoherence with records as it is mentioned earlier.

belong, is determined by the ‘basis’ we measure the records in the final time. *i.e.* we perform an actual physical measurement in the Copenhagen sense, in the final time to get one particular record (corresponding to the whole ‘generalized trajectory’ of our system). The basis we choose to measure the ‘record space’⁹ will single out a unique ‘preferred’ set of decoherent histories.

To get the relative frequency of these histories (corresponding to the probability of finding the one record or the other given a fixed basis), we repeat the experiment. Then, by making certain assumptions about these records, we recover the topological structure of the underlying spacetime. The assumptions are that each record consists of sub-records which we may identify as records of *events*.

Here we should stress that due to the above assumption that the records capture the spatiotemporal properties of the system and are thus records of events, the extended configuration space of the system is identified as the ‘*effective spacetime*.’ This identification is valid only under the above assumption about the nature of the records considered.

What is done, is that from a set of events, with no other structure presupposed (*a priori* imposed from outside the system), we end up with a causal set representing the discretized version of the *effective spacetime* the system in question lies. We will then proceed to consider the topology of this spacetime. We should stress here that the topology that we are speaking here and that we will use in the rest of the paper, concerns spatial topology, as it is usually understood. So in the case that we will consider a ‘spacelike’ surface, topology is understood as the spatial one (and observables of this will be things like the homology) while when we speak of ‘4-dimensional’ topology we will mean a series of ‘3-dimensional’ topologies that are ordered according to time (or more precisely a parameter-time). In the latter case we can also speak of transitions of one ‘3-topology’ to another.

The spacetime that we get will be an ‘*effective*’ one, and in a sense it accounts for certain properties of the Hamiltonian, such as interactions with other objects not controlled by the experimenter. For instance, the latter could be some kind of ‘repulsive’ field that prohibits the system to go somewhere (in a region of its configuration space), which can then be recovered as a hole (a dynamically inaccessible region) in that space. Note that since no force propagate faster than the speed of light, the causal structure will not be affected by any interactions and the set of events would give us the effective spacetime.

Therefore in our set up we may carry out our experiment sufficiently many times and we have access to the following two things: (i) the set of possible histories and (ii) the relative frequencies for each history to occur for every initial state. From this we recover the parameters of the experiment.

⁹That physically could correspond to an environment that produces decoherence for the system in question.

Furthermore, we assume that *the records capture the spatio-temporal properties* (of the system in focus). This means that *the histories are coarse-grained trajectories* of the system, belonging to a space whose topological properties we ultimately wish to deduce. We shall then claim that the whole concept of spacetime, as a background structure, does not make sense in finer-grained situations. In this way, all the histories are single-valued on our discretized version of ‘effective spacetime.’ One should note here that we may still have histories that have the particle in a superposition of different position eigenstates, but only if the latter are ‘finer’ than the degree of our coarse-graining. With the coarse-graining we effectively identify (*i.e.* we group into an ‘equivalence class’ of some sort) the points that we cannot distinguish operationally, with the resulting equivalence class of ‘*operationally indistinguishable points*’ corresponding to a ‘blown up,’ ‘fat point’ in our discretized version of ‘effective spacetime.’

1.3. Records and Sub-Records

A record of a trajectory, for example in a cloud chamber experiment (if the initial state is unknown) does not give us information about the direction the particle went. We can only deduce the order (up to an overall flip) because we assume certain things about the dynamics of the system and assume that the trajectory is continuous (and smooth in the metric of the given a priori background structure). In our case the background is not given and due to the restrictions of causality and having the set of possible alternative histories, we will deduce which events are close and therefore deduce the (causal) structure of the effective spacetime.

This relate to the general philosophical issue of how we understand from a frozen picture (final time record) the notion of change in the past (see also *e.g.*, Barbour, 1999). In particular in our operationalistic approach, we do not have access to any other information but the record, which as a frozen picture, does not have information about the order if no other assumptions about the dynamics are made. The whole thing would correspond in having a set of successive pictures each of which corresponds to a different possible trajectory (and NOT of different events, since each record is a whole history). Clearly, in each picture we would know the points the system crossed (‘sub-records’), but the order that it crossed them, would be unknown. In the photograph, the metric of the Euclidean space of the ‘photographic plate’ would give us already the order up to total direction flip, of the events which in this case would correspond to points the system visited. In our case we would have two complications. First there is no background structure to deduce which point is close spatially to which and therefore by continuity and smoothness of the trajectory to deduce the order of the events, and second we have more complex causal curves, *i.e.* not order but partially ordered set. We need to get some notion of proximity on these events based only on the records. We

will be describing two ways to do this in the paper, the one using the variation of records (see below) and the other using the partial order and the set of all possible different trajectories.

In the rest paper when we will refer to records, we will mean sub-records while for the full record we will refer as a history, causal chain or trajectory.

1.4. Effective Topology

In our approach, we consider the topology of the effective spacetime, which we derive from our observations. Thus we may or may not assume the existence of the spacetime with a certain topology. In either case we cannot determine this ‘*real*’ topology from our measurements and we therefore merely speak of *effective topology*—the topology of a model of spacetime which accords with our experiments and fits their outcomes.

Consider an example. Suppose we have derived a non-trivial topology for the ‘background’ spacetime—say, for instance, that it has a defect, such as a hole. This only indicates us that we have non-contractible loops, nothing more. Why these loops fail to be contractible—due to the existence of a ‘real hole,’ or because of, say, the presence of a potential barrier—such a question is, as a matter of principle, not verifiable within our approach.

1.5. Operationalistic Setup

Our approach is essentially *operationalistic*. The set of records, is regarded as the only source of information we possess about the system we wish to explore. The effective topology then refers to the configuration space/ effective spacetime of the system in question. In our tomographic approach, we are given the sets of observed histories together with their relative frequencies, from which then we reconstruct the parameters of the problem.

We assume that some of the records may be identified with particular events, *i.e.* spacetime ‘points.’ Furthermore, we claim that this is the only case we may speak of a background spacetime proper. That is, if we do *not* have access to events even in principle, we *cannot* speak about their support or their topological and causal nexus, as, say, in the causal set scenario (causet). Then, relative frequencies are recovered by repetition of the whole histories involved: by restarting the system in an identical environment and letting it evolve for the same amount of time.¹⁰ In our operationalistic (ultimately, relational-algebraic) view, the only way one can talk about some background structure such as ‘spacetime,’ is relative to something else. More precisely, we use our data (records) to (re)construct an ‘arena’ for a

¹⁰ From our vantage, ‘*history could in principle repeat itself*’ (pun intended).

particular subsystem of the universe that we are interested in, and it is *only* in this sense that we may speak of ‘spacetime.’

More precisely, we have

- (a) A system (call it ‘particle’), which is placed into an appropriate experimental environment, and we are able to repeat the experiment with *the same* initial conditions. In this way we get the relative frequencies of the records.

We may also vary the initial conditions of the system in question, leaving all the environment (and records) the same. For each initial condition of the system, we rerun the experiment. These first two steps give us the set of all possible histories (coarse-grained trajectories) of the particle, as well as their relative frequencies.

- (b) The space of records. It is a space of data resulting from controlled environment tampering with the system, and it is supposed to capture its spatiotemporal properties. Records are interpreted as *distinguishable* spatiotemporally, events. That is to say, that despite the fact that we do not know the structure of the set of records that corresponds to events, we can identify each record corresponding to a spacetime point as being different from the others. Thus, while we know nothing *a priori* about their causal or spatial (topological) ordering, events can be labelled so that we do not have identification problems.

We can vary each record corresponding to a particular event independently. The variation is in some sense small—this may be effectuated by a ‘small energy’ variation of the record. The latter is assumed to be small enough not to affect the ‘topology’ of the records (*i.e.* neighborhoods in the set of records remain the same). By ‘topology’ we mean a reticular structure associated with appropriate coarse-graining of a region of the effective spacetime we explore. The aforementioned variations give us the proximity relations between events in the classical case (Raptis *et al.*, 2005) and for the ‘statistical’ recovery of topology which will be described later.

Experiments are carried out repeatedly and multiply. We label the runs by initial conditions of the system, number of run and ‘positions’ of events.¹¹ Each run gives us a history, *i.e.* a sequence of causally related events that in the relativistic case will correspond to a causal chain.

The records correspond to different repetitions of the same experiment (of non-trivial temporal ‘width’), but could also correspond to simultaneous measurements of different systems (that have evolved) and where initially in the same state. One could imagine such a scenario considering the data from different angles of the CMB (Cosmic Microwave Background) presupposing isotropic universe.

¹¹ By this we mean whether or not we varied one record corresponding to an event.

To conclude, from our experiments we get the following information:

1. The set of histories of the system associated with a fixed set of initial conditions. We call this set of histories FIDUCIAL SET. Here we emphasize that these correspond to coarse-grained ‘trajectories’¹². We denote the **set of all histories** to be \mathcal{C} , while each history that is contained in it is denoted by C_i . Note that within these histories-‘trajectories’ the order of the events is not known. The set of all possible events, or else the set of ‘spacetime’ points will be denoted by \mathcal{P} .
2. The relative frequencies of outcome of these histories depending on the initial conditions. This is a function

$$f_j : \mathcal{C} \rightarrow [0, 1]$$

which gives the relative frequency of histories for each particular initial condition (corresponding to j^{th} initial state of the system).

3. The change in the relative frequencies when one event is varied. This is a function

$$f_j^p : \mathcal{C} \rightarrow [0, 1]$$

which is the *new* relative frequencies when the event p has been varied. This will lead us to the statistical way to get proximity relation between the points produced by the fiducial set of histories.

It is important to note that we already have the fiducial set of histories *before* we vary the records. This fiducial set of histories provides us the set on which the topology is imposed.

In Raptis *et al.* (2005) we used all that to get as much information about the underlining topological space as possible in the non relativistic case. More precisely, we were able to recover the number of components the effective spacetime had and the number of components a spatial surface had from purely algebraic considerations, not using the third of the above mentioned information. We were also able to recover the topology of the extended configuration space (corresponding to the background effective spacetime), using the proximity relation on spatial surfaces derived from the change of relative frequencies. The latter was named **statistical approach**.

2. RELATIVISTIC CASE

When we dwell on the relativistic case, the speed of the light is the upper bound c in the transmission of (material) information, this further restricts the set

¹²The inverted commas are added to the word ‘trajectories,’ since the space on which they are defined is not presupposed.

of possible histories-trajectories. The restriction is simply that they need to be causally related, which gives rise to a partial order in the set \mathcal{P} .

Here we should point out that our construction lies in the ‘organization’ of some records that are physically measured. This implies, that in some sense we do have a preferred frame, namely the one that the observer (us) that measures the records, lies. But what we wish to highlight is something different: once the records have been collected, we reconstruct an effective causal space (‘time-space’) structure, and only after this is done we can study its (again effective) transformation theory (relativity). The latter is already built into the ‘kinematical’ variation of records technology that we suggest.

Now we will first consider the case where we are given a partially ordered set as our effective spacetime. Later we will come back to the case where the relation between the elements of the set \mathcal{P} is unknown, but we do know the set of possible histories (causal curves) in the form of a covering of the set \mathcal{P} with subsets C^i . The set of all these is $\mathcal{C} = \{C^i\}$. In the sequel, we will restrict our attention to histories that are causal curves. Our considerations are similar to those that led us to consider trajectories in the classical case (see also comment in introduction before the records and sub-records paragraph). We should stress here, once more, that in the context of our set up the histories-chains are a covering with subsets of \mathcal{P} that within each of these subsets the order of the events is unknown.

2.1. Partially Ordered Sets

A partially ordered set, usually abbreviated as POSET, is a set \mathcal{P} endowed with a relation \preceq having the following properties:

- Reflexivity: $\forall p \in \mathcal{P} \quad p \preceq p$.
- Transitivity: $\forall p, q, r \in \mathcal{P} \quad p \preceq q, q \preceq r \Rightarrow p \preceq r$.
- Antisymmetry: $\forall p, q \in \mathcal{P} \quad p \preceq q, q \preceq p \Rightarrow p = q$.

A subset $C \subset \mathcal{P}$ is called a CHAIN (also known as a LINEARLY ordered subset) if any pair of its points is ordered: $\forall p, q \in C \quad p \preceq q$ or $q \preceq p$. In the sequel, we shall consider maximal (that is, inextensible) chains in \mathcal{P} and we will denote the set of all maximal chains by \mathcal{C} :

$$\mathcal{C} = \{ \text{maximal chains of } \mathcal{P} \} \tag{1}$$

In a similar way, we define an ANTICHAIN to be a subset S of \mathcal{P} such that no pair of its points is ordered: $\nexists p, q \in S \quad p \preceq q$. We shall need maximal antichains in \mathcal{P} , and denote the appropriate set by \mathcal{S} :

$$\mathcal{S} = \{ \text{maximal antichains of } \mathcal{P} \} \tag{2}$$

2.1.1. *Causets*

Discretized spacetimes with an ‘inherent’ causal structure are referred to as CAUSAL SETS, or causets for short. Causets are partially ordered, locally finite sets. The points of causets are thought of as spacetime points (:events). Local finiteness represents the (supposed!) fundamentally discrete nature of spacetime. It has been developed as a possible alternative to the spacetime continuum (:manifold) of General Relativity (Sorkin, 1991a,b; Zapatrin, 1993). Like in General Relativity, in every causet we can define both future $J^+(p)$ and past $J^-(p)$ cones for each of its events ($p \in \mathcal{P}$):

$$\begin{aligned}
 J^+(p) &= \{q \in \mathcal{P} \mid p \leq q\} \\
 J^-(p) &= \{r \in \mathcal{P} \mid r \leq p\}
 \end{aligned}
 \tag{3}$$

An element of a \mathcal{P} is said to be minimal if $J^-(p) = \{p\}$, and maximal when $J^+(p) = \{p\}$. The notions of future and past cones can be extended to subsets of \mathcal{P} . For $A \subseteq \mathcal{P}$

$$\begin{aligned}
 J^+(A) &= \{q \in \mathcal{P} \mid \exists a \in A \quad a \leq q\} \\
 J^-(A) &= \{r \in \mathcal{P} \mid \exists a \in A \quad r \leq a\}
 \end{aligned}$$

In terms of posets, the chains stand for causal curves, while the antichains are reticular analogues spatial (hyper)surfaces. A foliation \mathcal{F} is a partition of a causet \mathcal{P} into spatial surfaces (*i.e.* antichains) which respects the partial order \leq in \mathcal{P} , namely,

$$\forall A, B \in \mathcal{F} \quad A \cap J^+(B) \neq \emptyset \Rightarrow A \subseteq J^+(B)$$

Starting from this, we may introduce an ordering \sqsubseteq on \mathcal{F} ; namely, for $A, B \in \mathcal{F}$

$$A \sqsubseteq B \Leftrightarrow A \subseteq J^+(B)
 \tag{4}$$

A discrete analogue of a globally hyperbolic spacetime is a linearly foliable causet, *i.e.* when the order \sqsubseteq is linear (see definition above). It can be shown that any past finite causet (and this is the case we presently consider) admits a linear foliation, while this is not the case for a spacetime which admits closed timelike curves. The following construction proves this statement:

- $A_0 := \{\text{minimal elements of } \mathcal{P}\}$
- $A_1 := \{\text{minimal elements of } \mathcal{P} \setminus A_0\}$
- $A_k := \left\{ \text{minimal elements of } \mathcal{P} \setminus \left(\bigcup_{j=0}^{k-1} A_j \right) \right\}$

Here we should note the existence of one ‘preferred’ foliation in each past-finite causet, which is the one derived from the above mentioned procedure. In this foliation, each event is in the n th surface¹³ where n is the maximum number of steps to reach to the event by following a causal curve. So, the rôle of spacelike surfaces in our approach is played by antichains in \mathcal{P} that belong to \mathcal{F} .

2.1.2. Reconstruction of causal sets

As described above, our experiments provide us with causets that are only unstructured collections of points \mathcal{P} (:events), without telling us anything about their partial order. However, we know histories which are maximal causal chains (denote them $\mathcal{C} = \{C^i\}$).

For a time being, recall the classical case, where all histories that involved one event in every spatial surface, were possible. So we could have a history that in two consecutive instants have events infinitely ‘far’ spatially. Thus the information that two points belonging to two consecutive spatial surfaces could be in the same history added no information about their ‘spatial’ proximity and we were therefore unable to deduce more things about the topology other than the number of components. We were forced to use further measurements and recover the proximity with the statistical approach. In the relativistic case, in contrast, the upper limit in the transmission of information can provide us with extra information and we will be able to recover more things merely from the fiducial set.

Returning now to the relativistic case, we have a set and its covering by subsets. The reconstruction procedure looks like the following branching process (Breslav).

- Step 1. Pick a **maximal** collection of points $S^0 \in \mathcal{P}$ such that no pair of points $p, q \in S^0$ belong to a chain C^i . This S^0 will be the set of minimal elements. Assign $i := 1$.
- Step 2. Consider the set $\mathcal{P}^i = \mathcal{P} \setminus S^{i-1}$.
- Step 3. Pick a **maximal** collection of points $S^i \in \mathcal{P}^i$ such that no pair of points $p, q \in S^i$ belongs to a chain C^i (of the initial event set \mathcal{P} !). This S^i will be the second layer, if it exists, and is assigned $i := i + 1$. Then go to Step 2. If such S^i does not exist, the branch fails and one should restart from Step 1.
- Step 4. First check for non-appearance of extra chains. If it turns out that the foliation involved gives rise to a new chain, the branch is rejected. Then return to Step 1, restarting with a different maximal antichain. To see an example of the appearance of an ‘extra chain,’ see below.

¹³The antichains obtained in this way are not necessarily maximal in \mathcal{P} . For instance, in the example presented in Section 3 we have $A_4 = \{6\}$, which is not maximal in \mathcal{P} as it can be augmented to, say, $\{5, 6\}$.

Step 5. If the set \mathcal{P} is exhausted and all the causal chains can be reproduced without emergence of a ‘new’ one (see an example below), then the collection $\mathcal{S} = \{S^i\}$ forms the foliation of \mathcal{P} , the latter regarded as a causet proper.¹⁴

What we have effectively done in the foregoing is the following. We picked a partition of \mathcal{P} into anti-chains. To define the anti-chains we used the set of causal chains—histories. We then chose randomly an order on these anti-chains. After that, we checked that our construction did not produce any new histories (extra chains, in other words). If it did, we restarted the procedure.

2.2. Chains and Cones

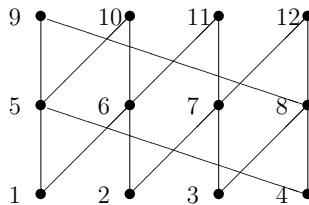
The set $\mathcal{C}(p)$ is the union of all maximal chains containing a point $p \in \mathcal{P}$. For an arbitrary point, we have $\mathcal{C}(p) = J^+(p) \cup J^-(p)$; thence, for minimal elements (for which $J^-(p) = \{p\}$)

$$\mathcal{C}(p) = J^+(p)$$

The way to derive histories is the following. We pick a point in S^0 and see which points are causally connected with it in S^1 . Then, we continue with the point we chose from S^1 and do the same with S^2 . Note that if we had chosen the correct foliation we would not have obtained new chains, because of causality’s transitivity.

2.3. An Example of ‘New’ Chain

Here we show how ‘extra chains,’ not existing in the initial poset, may emerge during the reconstruction procedure described above in Step 4. Consider the poset \mathcal{P}



¹⁴The end-product of this algorithm is guaranteed to be a foliation since all posets involved are foliable (see Section 2.1.1.).

and try to restore the order starting from {5, 6, 7, 8} as the set of minimal elements. So, if 5 is a minimal element, then $J^+(5) = C(5) = \{1, 4, 5, 9, 10\}$. Then take the element 1 (for which we deduced $5 < 1$). In the set $\mathcal{P} \setminus \{5, 6, 7, 8\}$, consider $J^+(1) = \{1, 9, 10, 11\}$, hence $1 < 11$. Thus, the chain {5, 1, 11} must exist, but actually it does not(!), therefore we reject the initial supposition that the antichain {5, 6, 7, 8} is minimal.

3. AMBIGUITIES IN ALGEBRAIC CAUSET CONSTRUCTION

Note that in the above way, we will eventually recover a foliation with some ambiguities, *i.e.* we will not get a unique partial order. The different causets we will get will be related to each other by some ‘symmetry’ transformations. One obvious would be an overall flip in direction. Another one would be related with points that exist in all histories and could be thought of as one-point spacelike surfaces. The order this ‘surface’ would exist, is ambiguous. To further classify these ambiguities, let us proceed first to some definitions.

Definition 1a: Let A be a subset of \mathcal{P} . We define the ‘initial surface’ of A , S_i^A to be the set of minimal points of the subset A when considered as a partially ordered set with respect to the partial order induced from \mathcal{P} .

Definition 1b: We also define the ‘final surface’ of A , S_f^A to be the set of maximal points of the subset A when considered as a partially ordered set with respect to the partial order induced from \mathcal{P} .

Definition 2: We define a point q in a partial order \mathcal{P} to cover $p \in \mathcal{P}$ to mean that q is in the future of p ($q \geq p$) and $\nexists r \in \mathcal{P} \mid q \geq r \geq p$.

Definition 3: Transitive closure of a point p in a partial order \mathcal{P} is the set of points $q \in \mathcal{P}$ such that there exists a sequence of chains $\{C_j\}^{15}$ with $p \in C_0$, $q \in C_n$ and $C_k \cap C_{k+1} \neq \emptyset \quad \forall k \in [0, n]$.

Definition 4: One-component subset, is a subset A of \mathcal{P} , that when consider as a partially order set from the induced from \mathcal{P} order, it has only one component, *i.e.* the transitive closure in A , (when considered as partial ordered set) of any point $p \in A$ is the set A itself.

One consequence of the above is that in a one-component subset A , every subset of the final surface, $D \subseteq S_f^A$, has in its past at least a point p that has in its future points in the complement of D in S_f^A , D^c . A similar statement holds for the initial surface S_i^A .

Definition 5: We define a subset A of \mathcal{P} to be ‘complete’ if $^{16} A = J^-(S_f^A) \cap J^+(S_i^A)$.

¹⁵ j going from 0 to n .

¹⁶ The following is equivalent with the condition: $\forall p, q \in A, \quad J^+(p) \cap J^-(q) \subseteq A$ and $J^-(p) \cap J^+(q) \subseteq A$.

Definition 6: We will call the subset A ‘information closed’ (abbreviated as *i.cl.*), if it has the following properties:

- (a) $\forall C_j \in \mathcal{C} \mid \exists p \in S_i^A \text{ and } p \in C_j \implies \exists r \in S_f^A \text{ and } r \in C_j$
- (b) $\forall C_j \in \mathcal{C} \mid \exists p \in S_i^A \text{ and } p \in C_j \iff \exists r \in S_f^A \text{ and } r \in C_j$

The above conditions means that any ‘ray’ (causal chain) that enters the initial surface will cross the final AND any ‘ray’ that crosses the final has also crossed the initial. This would mean that no ‘information’ from other places of \mathcal{P} enters or leave.

Note also that when the subset considered is a ‘complete’ one, then the following condition is equivalent with (a) and (b):

$$\left(J^-(S_f^A) \setminus J^-(S_i^A) \right) \cup S_i^A = \left(J^+(S_i^A) \setminus J^+(S_f^A) \right) \cup S_f^A = A$$

Otherwise this condition gives the completion of the subset A (*i.e.* the smallest complete subset that contains A).

Finally any p in a complete *i.cl.* subset that does not belong to the initial or final surface is covered only by points in the subset.¹⁷

Definition 7: We define a ‘flip’ of a subset A of a partial order \mathcal{P} to be a new partial order \mathcal{P}' having the same elements as before, and the relations between the points to be as follows:

- (a) All the points in $\mathcal{P}' \setminus A$ have between them the same relation as in $\mathcal{P} \setminus A$,
- (b) The relation of any point $p \in \mathcal{P}' \setminus A$ with any point $q \in A$ is the same as the relation of the same point p now belonging to $\mathcal{P} \setminus A$ with the point $q \in A$.
- (c) The relation of points $p, q \in A$ when seen as subset of \mathcal{P}' the new partial order, is the opposite of the relation between the same points p, q when seen as point belonging to a subset of the old partial order \mathcal{P} .

This condition already puts certain constrains, since the partial order is transitive, and we do not want to alter the relation of two points outside A and we also should not make any closed loop.

Note here that we could have considered a more ‘relaxed’ definition of ‘flipped’ subsets that we did not require the condition (b). In that case we would have allowed to flip the relation of some points in the subset with some other points that are outside imposing that the relation of the point outside the subset with other points outside, does not change. This is exactly as if we had included the point in the subset but required the condition (b) for flipping a subset to hold. This suggests

¹⁷That is true, since there is no ‘ray’ escaping the final surface (*i.cl.*) and there are no ‘holes’ since the subset is complete.

that we can get all the possible ‘inversions’ of subsets of the ‘weaker’ condition, by some ‘flips’ of the kind defined in definition 7 and therefore this definition is general enough.

Definition 8: A subset A of a partial order \mathcal{P} , is called ‘invertible,’ if we can ‘flip’ the overall order of the subset A without altering the set of ‘causal chains’ of \mathcal{P} .¹⁸

Theorem 1. A one-component subset A of \mathcal{P} is ‘invertible’ if and only if it is:

- (a) ‘Complete.’
- (b) Information closed.
- (c) If $p \in \mathcal{P}$ covers $q \in S_f^A \Rightarrow p$ covers $r, \forall r \in S_f^A$.
- (d) If $q \in S_i^A$ covers a point $p \in \mathcal{P}$ implies that all $r \in S_i^A$ covers p .

Proof: We will first show that if a subset A of \mathcal{P} obeys the conditions (a)–(d), implies that A is ‘invertible,’ by construction.

By the condition that the subset A is complete and information closed we know that the only ‘direct’ links between points of A and points of $\mathcal{P} \setminus A$ are those of $S_f^A = \{l_i\}$ with the points $\{p_i\}$ that cover them and those of $S_i^A = \{r_i\}$ with the points $\{q_i\}$ that are covered by the r_i ’s (see also final comment at definition 6). We could now ‘cut’ these direct links ‘invert’ the order of relations in subset A and then join back A in such a way that all the points of $S_i^A = \{r_i\}$ are covered by all the points $\{p_i\}$ and the points of $S_f^A = \{l_i\}$ cover all the points $\{q_i\}$. Condition (c) and (d) guarantee that this will not produce new chains, since all ‘sub-histories’ in A are linked with all the p_i ’s and q_i ’s. In this way we will end up with a partial order \mathcal{P}' that has the same set of chains with \mathcal{P} with the subset A having the opposite relations in \mathcal{P}' and the points in the rest set having the same relations between them and between them and points of A as required by definition 7.

We now proceed to prove the converse, showing that each of the conditions (a)–(d) are necessary conditions.

(a) If a subset A is not ‘complete,’ this means that: $\exists p \in \mathcal{P} \setminus A \mid r \succeq p \succeq q$ for $q, r \in A$. Inverting the relation between q and r implies that $q \succeq r$ which makes impossible for p to be in the future of q and in the past of r without creating a closed loop. So according to the condition (b) of definition 7 the subset A cannot be flipped.

(b) If a subset A is not information closed this means that there exists at least a ‘ray’ passing from the subset and either escaping the final or the initial surface. Let $p \in A$ be the last point in A of the escaping ray, and assume, for the moment, that the ray escapes to the future and $q \in \mathcal{P} \setminus A$ covers p . If the point p belongs to the final surface S_f^A we consider another ray, since this ones does not contradict

¹⁸This condition adds more restrictions to the allowed ‘flips.’

the definition of information closed (rays do escape from final surface to the future and from the initial to the past). There exists at least one chain containing p, q and no other point in the future of p in A . By inverting the subset A , every history containing p, q will also have points from what used to be the ‘future’ of p in A and therefore the chain that we mentioned before would not exist and the subset A is not ‘invertible.’ Note that if the ray escapes to the past, similar argument holds, considering the initial surface instead of the final and the past of p instead of the future.

(c) If the condition (c) did not hold, it would mean that there exists a point $p \in \mathcal{P}$ that covers a subset of the final surface $D \subseteq S_f^A$ and does not cover the complement of D in S_f^A, D^c . This would mean that either there doesn’t exist a chain including p, q_j where $q_j \in D^c$ or there doesn’t exist a chain with p, q_j and no other element of $\mathcal{P} \setminus A$ in between p and q_j . On the other hand, there exists a chain with p, r_i where $r_i \in D$ with no other element of $\mathcal{P} \setminus A$ in between p and r_i .

Inverting subset A , if we want to have a chain with p, r_i where $r_i \in D$ with no other element of $\mathcal{P} \setminus A$ in between p and r_i , we will have to create at least one chain including p and q_j with no other element in between. This is due to the fact that subset A being one-component subset, it necessary has the property that in the past of points $r_i \in D$ there exists at least one point that has in its future a point in D^c (see comment after definition 4). Thus inverting the relations in the subset will bring to ‘same’ fate the point in D and the point in D^c that have in their past the point connecting them. We would have thus, created a new chain and the subset A would not be ‘invertible.’ Therefore it has to obey condition (c).

(d) This can be proven similarly to (c). Note that if we invert all the relations in \mathcal{P} condition (d) becomes condition (c) and since the set of histories is clearly not affected by an overall flip this condition should also hold.

This completes the proof. □

A direct consequence of the above theorem, is that if a subset is information closed and $(S_i^A \subseteq S_i^{\mathcal{P}}$ or $S_i^A = \{p\}$ for some $p \in \mathcal{P}$) AND $(S_f^A \subseteq S_f^{\mathcal{P}}$ or $S_f^A = \{q\}$ for some $q \in \mathcal{P}$) then the subset is invertible. This is due to the fact that the previous condition is just a special case of the theorem.

If we want to consider a subset that has more than one components, we treat each component separately.

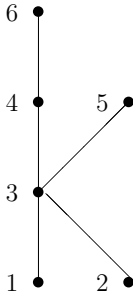
We furthermore speculate, that any ambiguity in the causet construction of Section 2.1.2. is due to some ambiguity of the direction of some subsets. This is natural to assume, since the information about the direction is not given from the set of histories-causal chains.

Conjecture 1. *We can transform any partial order to another with the same set of chains (when considered as subsets with no order) by some combinations of flips of one-component subsets.*

Here we should also note that a Minkowski space (where information ‘spreads’ in space from every point) or actually in any space having that feature, there wouldn’t be neither any non-trivial *i.cl.* subsets nor any subset obeying conditions (c) and (d), and furthermore we suspect (if conjecture 0 is true) that we wouldn’t have any ambiguity apart from the overall flip or else ‘time-reversal.’¹⁹

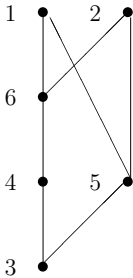
Let us now explore an example below to demonstrate the above.

An Example. Consider the poset \mathcal{P} :



Assume we just have the set of chains-histories. Try to restore the partial order, starting with $\{3\}$ as the set of minimal elements S^0 . Set as second layer the surface $S^1 = \{4, 5\}$, as third $S^2 = \{6\}$, and last, the set $S^3 = \{1, 2\}$. Now $J^+(3) = \{1, 2, 4, 5, 6\}$. Choose one of the points in the future of 3 that are in S^1 —say for example, choose 4 so that we have $3 \preceq 4$. Pick one point in S^2 that is causally connected to 4. This is the point 6. Repeat this for the last layer and end up with the two histories $\{3, 4, 6, 1\}$ and $\{3, 4, 6, 2\}$, that exist. Now choose the element 5 from S^1 . Then, there does not exist any point in S^2 that belongs to the same history with 5 (since 6 is the only element there, and it is not connected to 5). We continue with the next surface S^3 . With the procedure described we have recovered the following histories: $\{3, 4, 6, 1\}$, $\{3, 4, 6, 2\}$, $\{3, 5, 2\}$, and $\{3, 5, 1\}$. We have thus recovered all the histories, no matter that it is *not* our initial causet.

‘Wrong’ poset \mathcal{P}' :



¹⁹Note that the general features described above are not necessarily satisfied by our operationalistic ‘effective’ spacetime.

The important point of this ambiguity is that the surface $\{4, 5\}$ is before the $\{1, 2\}$ while normally should be the other way round.

From our initial causet we could end up to the one just described, by some transformation related with the flipping of some subsets that are ‘invertible’. First we have an overall flip to end up with $S^0 = \{5, 6\}$, $S^1 = \{4\}$, $S^2 = \{3\}$ and $S^3 = \{1, 2\}$. Then we consider the subset $A = \{3, 4, 5, 6\}$ that is ‘complete,’ *i.cl.* and obeys conditions (c) and (d). This is true, since 6 and 5 have no point covering them in the past, and point 3 being one point surface and also the final surface of subset A is covered by 1 and 2 and obeys (c). This, by theorem 0, means that it is ‘invertible’. We flip the subset A and end up with $S^0 = \{3\}$, $S^1 = \{4, 5\}$, $S^2 = \{6\}$ and $S^3 = \{1, 2\}$ which is the false causet that we got previously being consistent with our fiducial set of histories.

We should note here, that if instead of the subset A we took subset $B = \{3, 4, 5\}$ it would still be *i.cl.* but the condition (d) would NOT be satisfied ($S_i^A = \{4, 5\}$ and point 6 is covered by point 4 but not by point 5). The subset would still be in some sense invertible (not according to definition 7, but with the weakened definition 7 dropping condition (b)). We could put 6 in the future of the subset and join it with point 4, while we could keep the links of 6 as they were (1 and 2 covering 6) and then link 5 to 1 and 2. This would have altered the relation of point 6 that is not in the subset under consideration B, with points in the subset. This would give us $S^0 = \{3\}$, $S^1 = \{4, 5\}$, $S^2 = \{6\}$ and $S^3 = \{1, 2\}$ which is the ‘false’ causet we got. The important point here is that there isn’t a need to weaken the definition of ‘flips’ to allow such inversions, since we could get this by just containing the point 6 in the subset, to get subset A that gives the same result (see also comment after definition 7).

The bottom-line is that ambiguities that cannot be resolved by knowing all the histories are ‘intrinsic,’ and there is no physical argument for us to believe that we are in the one or the other causet. We could either say that we are in a superposition of causets (under the proviso that it is possible with further ‘measurements’ to determine which one we are in), or that these causets are operationally equivalent (:indistinguishable).

In our operationalistic approach, we may claim that if from our measurements we cannot distinguish sharply a causet that is in force, then our system is most probably described by a *superposition* of causets. This is in accordance with the idea that when a measurement is made, the state ‘reduces’ to the projection on the total subspace that we measured, rather than to the projection to a particular one-dimensional subspace. The other possibility is to regard the ‘physical’ states as being *equivalence classes* of causets with the equivalence relation being the existence of the same set of histories realizing them. In the later case though, given the fact that we expect the topology of those causets to be non-homeomorphic, it would be difficult to make any meaningful statement about the topology of the corresponding effective spacetime (see also Section 3.4.).

We have showed how to reconstruct the partial order of the effective spacetime in the relativistic case, in a combinatorial way and abide by (*i.e.* they can be checked according to) the following CONSISTENCY PRINCIPLE:

Choose a partial order \preceq on the set \mathcal{P} of events. Then we make a working hypothesis: ‘the partial order \preceq does not contradict our experiments.’ To accept or reject this hypothesis, we just build the set of all maximal chains $\mathcal{C}(\mathcal{P}, \preceq)$. Then, if $\mathcal{C}(\mathcal{P}, \preceq) = \mathcal{C}$, we accept the hypothesis; otherwise we reject it.²⁰ Note that the existence of a new chain, immediately contradicts this consistency criterion.

At this stage we have the causet associated with our laboratory.

3. RECOVERING THE TOPOLOGY: STATISTICAL VS ALGEBRAIC APPROACH

So far we have only used the set of all histories, while the relative frequencies have not yet been used. We shall now consider ways to recover some topology on this causet. Here we should remind the reader that when we speak of topology, we mean the ‘spatial’ topology in the way that is usually understood. Observables of this would be things like the homology or other topological invariants. When we will speak of the topology of the spacetime, we will mean topology of ‘3-dimensional’ spatial surfaces patched together according to an ordering. This ordering is according to the, (unphysical) parameter-time.

There are two ways to recover the topology. The first one is to vary the records as it was done earlier, in the classical case (Raptis *et al.*, 2005) and we will call it the **statistical** way of recovering topology while the second uses merely the derived causet as its only source of information and will be referred to as **algebraic** way of recovering topology.

3.1. Statistical Approach

We have the relative frequencies of each history C_i with initial condition ‘ j ’, labelled $f_j(C_i)$, and the relative frequencies having varied point ‘ p ’, labelled $f_j^p(C_i)$ (see final part in Introduction). We then take a small positive number $\epsilon \ll 1$. We define another function, the difference function, as follows:

$$\delta_j^p : \mathcal{C} \rightarrow [0, 1] : | f_j(C_i) - f_j^p(C_i) | \tag{5}$$

We then consider all the points belonging to the histories $C_i \in \mathcal{C}$ that $\delta_j^p(C_i) > \epsilon$. We name them j -neighbors of p . Physically we assumed that the relative frequencies of histories containing points close to the one we vary, will alter more

²⁰ Generically, the causal order is reconstructed up to the ambiguities described above. The corresponding topologies are in general different (non-homeomorphic).

than histories containing points only far from the point in question (spatially and temporally). So we have:

$$q \in N_j^p \implies \exists q \in C_i, C_i \in \mathcal{C} \mid \delta_j^p(C_i) > \epsilon$$

We then consider different initial conditions ‘ j ’ and we group all the neighbors together to form the neighbors of ‘ p ’, N^p .

$$q \in N^p \implies \exists j \mid q \in N_j^p$$

We define spatial neighbors of the point ‘ p ’ those points that are in N^p but do not belong to the any history containing ‘ p ’.

$$SN^p \mid q \in [N^p \setminus \cup_i C_i], \quad p \in C_i \quad \forall i$$

By repeating this for every point in each of one spacelike surface we may recover the proximity and therefore the topology of this slice in the usual way- *e.g.*, as it is done in metric spaces.

We will have obtained the topology of each spatial components. We can then choose an arbitrary partitioning of these slices to get the total ‘4-dimensional case’ where we will be able to see transitions from different topologies²¹. We then check that we do not have contradiction. This contradiction could be due to, for example, some event being affected by a change in an event to its future rather than to its past (‘advanced’ and ‘retarded’ contradiction, respectively). If a contradiction arises, we pick another ‘partitioning,’ so on and so forth, until the correct one is obtained. In this way, previous ambiguities in the causet construction, such as those related with an overall flip would be resolved. So in the previous example in section 3. the ambiguity would be resolved, since it contained an overall flip. In other, limited, cases, we would still have ambiguities. In particular the order of two points p, q that belong to an ‘invertible’ subset A of \mathcal{P} that consists of a single chain (*i.e.* a chain that is *i.cl.* and ‘complete’) would still be ambiguous if $p, q \notin S_f^A$. If they were in the final surface we could see that ‘varying’ them affected the relative frequencies of the next surface points (provided it is not a single one). If none of them were in the final surface, their order would remain ambiguous.

3.2. Algebraic Approach

We will assume for the moment that we have a unique unambiguous causet. In this case there exist already certain way to speak about topology (and certain geometrical properties furthermore).

²¹ We consider ‘effective spacetime’ and thus, we are expected to see topology changes.

Following (Brightwell and Gregory, 1991) we may define some notion of distance for timelike separated events, say $p \preceq q$ to be the number of maximum steps in the partial order one needs to travel to go from the one, p to the other, q . This would correspond to ‘proper time.’ We then proceed to define distance of two points being spacelike separated by considering the following. The only way for an ‘inertial’ observer in one point to know about its distance to another that is spacelike separated is by considering standard clocks and light beams. We would be therefore interested in the distance of a point from a geodesic corresponding to a history. We consider a point x and a geodesic C such that w and z are points of C such that $w \preceq x \preceq z$. For point x , let $l(x)$ be the highest point in C which is below x , and $u(x)$ the lowest point of C that is above x . Then $d_s(x, C) = d(l(x), u(x))/2$ where $d(., .)$ is the proper time.

Using those concepts one may define neighborhoods on the spatial surface by choosing a particular distance around every point (c.f. balls in usual metric spaces).

Alternatively one could follow (Major *et al.*, 2005) and thicken every anti-chain by considering the immediate future. Then use this to define some ‘shadows’ on the initial anti-chain, that would group together points to form a finite cover of this set, to intersecting subsets. The width of the thickening should be suitably tuned, to be big enough to capture global properties, but not too big in order to ‘identify’ correctly the neighborhoods and not to get to trivially intersecting cases (where all points are in the neighborhood of all). In Major *et al.* (2005) they proceed using simplicial complexes and ‘nerves’ to get the homology of the anti-chain when considered as an approximation of a spacelike surface of a manifold.

In both of these cases we get a cover of the anti-chain with subsets corresponding to intersecting neighborhoods. A way to define a topology on the anti-chain that would capture the spatial topological properties of a manifold that would be the approximation of the causet (giving fundamental status to the causet), is the following. We consider the subsets and their intersections and make a partial order of all these (neighborhoods and intersections) where the order is set inclusion. Note that this partial order is completely different that the causal partial order of our initial causet. From this new partial order we may consider the Alexandrov topology that would give us some ‘spatial topology’ on the spacelike surface in question. The Alexandrov topology on a partial order, is defined to be the topology where the open sets are the past sets of the partial order.

$$S \subseteq X : \quad \forall \quad x, y \in X, x \in S \quad \text{and} \quad y \preceq x \rightarrow y \in S$$

Where X is the partial order, and S are the open subsets. To calculate other properties, such as homology, we need to resort to simplicial complexes as in Major *et al.* (2005).

In both cases, we may define a topology of a spatial surface of a particular causet (if the causet is thought as a faithful approximation of a continuous manifold). To get a ‘4-dimensional’ topology (*i.e.* including the ‘time’ dimension or else considering Lorenzian rather than Euclidian manifold) we pack the slices according to the parameter time.

What affects the result is

- The choice of the causet structure (in case it is ambiguous).
- The choice of slice (*i.e.* the way we ‘foliated’ the causet in anti-chains).
- The choice of the size of the ‘balls’ used to define the neighborhoods or the ‘thickness’ of the slice, since those would affect the spatial topology.

In summa, in the ‘algebraic approach,’ we have arrived at a possible topology for our effective spacetime and, perhaps more importantly, for doing this we have only used the set of different histories.

3.3. Comparing the Approaches

We shall now compare the two aforementioned ways of recovering topology. Possible disagreements between them could stem from the following:

- (i) The variation of the records was not ‘small enough,’ so that the deduced topology does not correspond to the initial one, which means that the first way failed.
- (ii) The causet we chose is not the ‘real’ one and one of the ensuing ambiguities has possibly misled us, so that again the first construction has failed. Note that had we considered all the possible causets consistent with our data, we would find that one of them agrees with the causet derived from the first way, unless the first way failed due to the reason mentioned above.
- (iii) Finally, the two ways of drawing the proximity relation may intrinsically disagree, with the first way, thus failing to identify the ‘real’ nearest neighbor. This could also indicate the incompleteness of our model of the experiment *e.g.*, that the records we had, did not correspond to events.

These considerations rest on that from our causet reconstruction, we had a unique unambiguous causet (up to a total time flip); or, if more than one, that all resulted in homeomorphic topologies. In the general case, it seems that we need to fix the interpretation first. We can claim two things:

- (i) The state of the system is in superposition of different topologies, one corresponding to each possible causet. Further measurements that are made

by varying the records will result in a reduction of states. The probabilities for different causets (amplitudes in the superpositions) could be recovered if we repeated many times the whole procedure of varying the records. If the procedure always yields a particular causet, we may conclude that the state was in that ‘eigenstate’ from the beginning, and that it was us, that did not have access to the records. Here, our failure to identify the correct state was due to the fact that we were missing some information, namely, the results of the measurements associated with the variation of the records. We could therefore conclude that this failure was basically an epistemic one, due to some kind of ‘classical indeterminacy’ see also Raptis *et al.* (2005).

- (ii) The state of the system is the equivalence class of different causets, with equivalence relation being the possession of same causal curves. In this context we cannot talk about the topology of the equivalence class if there exist non-homeomorphic topologies in the same equivalence class.

3.4. Further Discussion

If we take the point of view that the causet structure derived from the set of histories is the best description we have for the system and give ontological status to all the possible different causets, we may as well enquire whether further measurements from us may ‘reduce’ the state to one particular causet (or to some equivalence class thereof). We would then be talking of a superposition of different causets. A way to handle these ‘quantized causets’ is by considering their incidence algebras as in Raptis (2000). It should be emphasized that if the variations are indeed small, then the derived causet should just be one of the possible causets derived without the variation.

The aforementioned method of determining the proximity by varying the records could be viewed as a set of extra measurements that restrict the class of possible causets—in a sense, as determining the ‘actual’ causet. Different procedures of deriving the proximity will of course ‘favor’ different causets. Furthermore, allowing to vary the records lifts most ambiguities, including the one involving an overall flip. If all procedures of obtaining the proximity result to the same causet, we may as well say that the structure of the effective spacetime of the system was determined, but it was us that did not have access to the records.

Since we are talking about an actual physical system in an actual lab, one would not expect that the system actually ‘experiences’ such topological transitions. We could though imagine the following Schrödinger’s cat type of gedanken experiment. We have a box with a particle inside. It is separated into two pieces, and whether the wall between them falls or not depends on some spin-half particle that is in a state of superposition. If we do not have access to that particle, and we

repeat many times the experiment, getting both topologies is an actual possibility (pun intended). Trying to determine which of the two is the ‘correct’ one would be equivalent to measuring the spin-half particle, and that would then give a definite answer.

Finally, the other way of taking ‘seriously’ all the possible causets is to consider the physical states as being equivalence classes of causets, related to a particular set of records (or else, to a particular state of the record space). Then, small record variations would merely pick one representative of our physical state. Further investigation is needed to establish what happens if bigger variations are allowed that would move from one equivalence class to another. What remains to be considered, in this case, is which of these classes of causets are close to which. To do so, we would still have to define a notion of small variation that would give us relative frequencies (however, the variations should be big enough to move us out of the equivalence class we happen to be, and into another).

4. CONCLUSIONS

Let us summarize what we have done. We have a laboratory in which we explore a physical system whose effective spacetime, the ‘arena’ it lies, is unknown. We are able to run the experiments sufficiently many times, either by leaving the initial conditions unchanged, or by varying them. We also have another physical system, whose configuration space is coined RECORD SPACE. In particular, we require from the record space to capture the ‘spatio-temporal’ properties of the system. We therefore have a set of records for events.

We made the assumption that these records correspond to spacetime points.²² The assumption we made (about the records) will then help us organize the information we have in the present, as something that was a history. We will be dealing with, in some sense, a ‘timeless’ theory, since time would emerge merely as a better way to organize present data. To paraphrase Wheeler (in his delayed choice experiments) (Wheeler, 1990),²³ ‘events’ become events when some records are observed and when, by our (‘delayed’) choice, those records are identified as events.

After multiple runs, we have a set of protocols (data-sheets). Each protocol tells us which events occurred within a particular experiment but it does not tell us in which order the events occurred. This set of events is referred to as a history which, in this context, correspond merely to a coarse grained ‘trajectory’ or else to a chain in a partial order that corresponds to the effective spacetime. When the initial conditions remain unchanged, the arising set of histories is treated as a decohering set.

²² More precisely, ‘effective’ spacetime points, since we cannot have directly access to the ‘real’ spacetime, if this thing exists.

²³ “No phenomenon is a phenomenon unless it is an observed phenomenon.”

More precisely, as a result of our observations, we have histories and, in addition, their relative frequencies. This primary set of histories we call FIDUCIAL SET. This corresponds to both the sets of all possible events \mathcal{P} and of all possible histories C .

Using the fact that there are restrictions to the set of possible histories due to causality, we proceed in Section 2.1.2 to obtain a partially ordered set corresponding to the spacetime (causal set). Due to our operationalistic methods, the causet is recovered up to certain ambiguities (Section 3).

Here we should point out that the above procedure accounts for the following mathematical task. Deriving a partial order given the set of possible chains as mere subsets of the total partial order, *i.e.* without having the order of points in each of these chains. This derivation is not unique and in Section 3. we classified the possible ambiguities.

We then considered two ways of recovering topology. The first one involves some extra measurements, namely, varying the records to get proximity and is similar to the one consider in the previous paper Raptis *et al.* (2005).

pt The second relies completely on the derived causet. We referred to some work other work (Brightwell and Gregory, 1991; Major *et al.*, 2005) on how to get topology from a given causet in Section 3.2. One could use any scheme for deriving topology from a causet, and the rest of the discussion would remain the same.

Since the above construction does not in general conclude to a unique causet, we need some ‘interpretation’ of the derived causet before we can compare the different ways of deriving topology. We may treat the possible causets as belonging to one equivalence class and consider this equivalence class as a physical state. This would run into problems if we would like to talk about topology, if the algebraically derived topologies for different causets in the same equivalence class are not homeomorphic.

Another way to interpret the set of all consistent with our histories causets, is to assume that the system we consider ‘lives’ in a superposition of different effective spacetimes, where each of the terms in the superposition, may have different topology. In this case, we would consider the variation of the records done in the ‘statistical’ way of recovering the topology, as a set of further measurements, causing the state to ‘reduce’ to a particular spacetime with its associated topology.

As a final point we would like to emphasize once again that we recover histories *operationalistically*. The record space is *the only* source of information we possess about the system we explore. The effective topology is then regarded as the ‘best possible’ (:as realistic, or as pragmatic a) picture of the actual background spacetime of the system in focus as one can acquire from her ‘experimental intercourse’ with it.

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